For all questions, answer choice (E) NOTA *means that none of the given answers is correct. Good luck!*

1. Goats have four legs, while gorillas have only two. At the Goats-and-Gorillas party (which only goats and gorillas are invited to), there are 24 animals and 60 legs. How many goats are at the party?

(A) 6 (B) 8 (C) 9 (D) 12 (E) NOTA

2. It is important to understand the relationship between degrees and radians. For example, $90°$ is equivalent to $\frac{\pi}{2}$ radians. Which of the following is equivalent to 30 radians?

(A) $\frac{\pi}{6}^{\circ}$ (B) $\frac{\pi}{3}^{\circ}$ (C) $\frac{5400}{\pi}^{\circ}$ (D) $\frac{10800}{\pi}^{\circ}$ (E) NOTA

3. Suppose $0 < \theta < \frac{\pi}{2}$ and $\ln(\sin\theta) - \ln(\cos\theta) = 1$. Which of the following inequalities is true?

(A) $0 < \theta < \frac{\pi}{6}$ (B) $\frac{\pi}{6} < \theta < \frac{\pi}{4}$ (C) $\frac{\pi}{4} < \theta < \frac{\pi}{3}$ (D) $\frac{\pi}{3} < \theta < \frac{\pi}{2}$ (E) NOTA

4. How many positive integers less than 1000 are divisible by 3 or 5?

(A) 401 (B) 466 (C) 467 (D) 532 (E) NOTA

5. Compute $\displaystyle\sum_{n=0}^{89} \cos\left((2n+1)^{\circ}\right)$.

(A) 1 (B) 2 (C) 44 (D) 45 (E) NOTA

6. Find the sum of all $x > 0$ that satisfy the equation $x^{\sqrt{x}} = x\sqrt{x}$.

(A) $\frac{9}{4}$ (B) $\frac{13}{4}$ (C) $\frac{7}{2}$ (D) $\frac{9}{2}$ (E) NOTA

7. Find the sum of the solutions $0 \le \theta \le 2\pi$ that are roots of $f(\theta) = \theta + \theta\cos\theta$.

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$ (E) NOTA

8. If the solutions for x to $f(x) = 0$ are 7 and 1, what is the sum solutions for x to $f\left(\frac{x}{3} - 1\right) = 0$?

(A) $\frac{2}{3}$ (B) 2 (C) $\frac{10}{3}$ (D) 30 (E) NOTA

9. Let $x_1, x_2, \ldots, x_{2015}$ be the unique solutions to $x^{2015} = 1$. Compute $x_1^2 + x_2^2 + \ldots + x_{2015}^2$.

(A) -1 (B) 0 (C) 1 (D) 2 (E) NOTA

10. Patrick and Chris have the following conversation about their favorite numbers:

PATRICK: My favorite number is an integer $P > 1$. It has the interesting property that $\sqrt[6]{P}$ is an integer.

CHRIS: My favorite number, C, isn't a real number! It has an interesting property, though: $C^{2015} = 1$.

PATRICK: Well, that doesn't help me much! I can think of more than P numbers that could be the value of C.

CHRIS: In that case, I now know the value of C^P.

Assuming Patrick and Chris make true/perfectly rational statements, which of the following values is equal to 1?

(A) C^3 (B) C^5 (C) C^{13} (D) C^{31} (E) NOTA

11. You've made it to page two; congratulations! Compute $\log_2(9) \cdot \log_3(4)$.

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4 (E) NOTA

12. Compute $\sin\left(\arcsin\left(\frac{1}{3}\right) + \arccos\left(\frac{1}{3}\right)\right)$, where the inverse trig functions are assumed to have their traditional restricted ranges.

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

13. What is the area of the triangle whose vertices are given by the polar coordinates $(0,0)$, $\left(1, \frac{\pi}{3}\right)$, and $\left(2, \frac{\pi}{6}\right)$?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) NOTA

14. For real numbers x, what is the maximum possible value of $3x - x^2$?

(A) $\frac{3}{4}$ (B) $\frac{9}{4}$ (C) $\frac{3}{2}$ (D) 3 (E) NOTA

15. Compute the area of the region in the (x, y)-plane that satisfies $\frac{x^2}{36} + \frac{y^2}{16} \leq 4$.

(A) 18π (B) 36π (C) 72π (D) 144π (E) NOTA

16. Suppose $0 \leq x \leq \frac{\pi}{2}$ and $\sec^2(x) + \tan^2(x) = 1 + \cos^2(x)$. What is $\sin^2(x)$?

(A) $2 - \sqrt{3}$ (B) $\sqrt{2} - 1$ (C) $\sqrt{3} - 1$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

17. Suppose that s and t are complex numbers such that $|s| = |t| = |s - t| = 1$. Compute $|s + t|$.

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 (E) NOTA

18. The result of the "Basel problem" is quite beautiful:

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{6} \cdot \pi^2.$$

Now, let $S = \{1, 2, 4, 5, 7, 8, 10, \ldots\}$ be the set of positive integers which are *not* divisible by 3. With the result of the Basel problem in mind, one can show that

$$\sum_{s \in S} \frac{1}{s^2} = \frac{a}{b} \cdot \pi^2$$

where a and b are relatively prime positive integers. Compute $a + b$.

(A) 10 (B) 13 (C) 19 (D) 31 (E) NOTA

19. Suppose that $0 < x < \frac{\pi}{2}$ and $\sin(x) = \sqrt{\cos(x)}$. Compute $\cos(x)$.

(A) $\frac{1}{4}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{\sqrt{5}-1}{2}$ (D) $\frac{\sqrt{5}+1}{4}$ (E) NOTA

20. What is the period of $f(x) = \sin(x) + \cos\left(x + \frac{\pi}{6}\right)$?

(A) π (B) 2π (C) 3π (D) 6π (E) NOTA

21. Eli and Allison are playing a game. Eli goes first, and he has to say a positive integer less than or equal to 16. Then, Allison must add a positive integer less than or equal to 16 to Eli's number, at which point Eli must add a positive integer less than or equal to 16 to Allison's number, and so on. The winner is whomever says the number 2015. What number must Eli say first to ensure that he will win the game if both players play optimally?

(A) 9 (B) 11 (C) 13 (D) 15 (E) NOTA

22. The circle $x^2 + y^2 = 4$ is tangent to a line ℓ with slope $\frac{1}{3}$ at a point on the circle in the fourth quadrant. Find the x-coordinate of the point where ℓ intersects the x-axis.

(A) 6 (B) $2\sqrt{10}$ (C) $4\sqrt{3}$ (D) 10 (E) NOTA

23. How many of the following four functions are even?

I. $a(x) = |x-2|$ **II.** $b(x) = \sin\left(x + \frac{\pi}{2}\right)$ **III.** $c(x) = \frac{e^x}{\cos(x)}$ **IV.** $d(x) = 3x$

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

24. What is the constant term in the expansion of $\left(2x - \dfrac{3}{x^2}\right)^6$?

(A) 144 (B) 720 (C) 1080 (D) 2160 (E) NOTA

25. Pacman (who travels at a constant speed of 8 units/second) is on the number line at 0 with two ghosts in pursuit: Blinky is at 100, while Inky is at -100. Blinky chases Pacman at a constant speed of 1 unit/second, while Inky chases Pacman at a constant speed of 5 units/second. Pacman travels in a straight line toward Blinky, and as soon as he reaches Blinky, he immediately turns around and travels in a straight line toward Inky. As soon as he reaches Inky, Pacman immediately turns around and travels in a straight line toward Blinky, and so on. Pacman will be eaten when he is eventually at the same location as Blinky and Inky. What is the total distance that Pacman will travel before he is eaten, rounded to the nearest 50 units?

(A) 200 (B) 250 (C) 300 (D) 350 (E) NOTA

26. Compute $\displaystyle\sum_{n=1}^{2015} \frac{1}{\sqrt{n} + \sqrt{n+1}}$, rounded to the nearest integer.

(A) 42 (B) 43 (C) 44 (D) 45 (E) NOTA

27. If x and y are positive integers greater than 1 such that $\log_{10}(x) = 2\log_{10}(y)$, what is $\log_y(x)$?

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2 (E) NOTA

28. How many unique, non-degenerate triangles exist with one side of length 3, another side of length 4, and some angle measuring $60°$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

29. Daniel has 3 coins: two are fair, standard coins (50% heads, 50% tails), while one has heads on both sides. Daniel chooses one of these three coins randomly (with each coin equally likely to be chosen) and flips it twice. Given that both flips are heads, what is the probability that Daniel has chosen the coin which has heads on both sides?

(A) $\frac{1}{2}$ (B) $\frac{5}{9}$ (C) $\frac{3}{5}$ (D) $\frac{5}{6}$ (E) NOTA

30. In Calculus[1], you will learn that

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \quad \text{and} \quad \sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}.$$

Recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$. For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\tan(x) = \displaystyle\sum_{k=0}^{\infty} c_k x^k$ where $c_0, c_1, c_2, \ldots$ are constants. Find c_5.

(A) $\frac{1}{8}$ (B) $\frac{7}{60}$ (C) $\frac{2}{15}$ (D) $\frac{13}{60}$ (E) NOTA

[1] We take $0^0 = 1$ in this context, so $\cos(0) = 1$.